Math 2326 - Introduction to Abstract Mathematics
Assignment 5 - Due Wednesday, January 30

In Problems 19 through 21, use proof by contradiction or contrapositive to establish the given statement.

Problem 19:

Let $a, b \in \mathbb{Z}$. Prove that if $a \neq 0, \pm 1$ then a cannot divide both b and $b + 1$.

Problem 20:

If $f(x) = x^3 + x + 1$ then the equation $f(x) = 0$ has exactly one real solution. [*Hint:* You may want to review the Intermediate and Mean Value Theorems from Calculus I.]

Problem 21:

Let $x, y, z \in \mathbb{Q}$. If $x + y \geq z$ then $x \geq z/2$ or $y \geq z/2$.

Problem 22:

Consider the following proposition:

Proposition. *For every real number x, $x^2 \geq 0$.*

What's wrong with the following proof of this proposition?

Proof. Suppose not. Then for every real number x, $x^2 < 0$. In particular, plugging in $x = 3$ we would get $9 < 0$, which is clearly false. This contradiction shows that for every number x, $x^2 \geq 0$. □