Math 2326 - Introduction to Abstract Mathematics Assignment 9 - Due Friday, February 8

Problems 35, 36: Write up solutions to two of the in-class induction exercises that you *did not* work on during Wednesday's lecture period. For your convenience these problems are restated below.

Exercise 1. Use the product rule for derivatives and the fact that $\frac{d}{dx}(x) = 1$ to prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all $n \in \mathbb{N}$.

Exercise 2. Suppose $p \in \mathbb{N}$ and f is a function that is p-times differentiable which satisfies $f^{(p)} = f$. Prove that for all $n \in \mathbb{N}$, f is n-times differentiable and that $f^{(n)}$ is one of the functions $f, f', f'', \ldots, f^{(p-1)}$.

Exercise 3. Let $k \in \mathbb{N}$ and let p(x) be a polynomial of degree k with real coefficients. Prove that for all $n \in \mathbb{N}$, the *n*th derivative of $p(x)e^x$ is of the form $q_n(x)e^x$ where $q_n(x)$ is a polynomial of degree k.

Exercise 4. Prove that every nonempty subset of \mathbb{N} has a least element.