Intro to Abstract Math
Fall 2009

Review for Midterm 1

Exercise 1. Define the following terms: *statement, converse, contrapositive, a divides b, subset, set equality, intersection, union, complement, (set) difference, empty set, disjoint, power set, (set) product, relation, equivalence relation.*

Exercise 2. Study all of the theorems and proofs given in class and on the homework.

Exercise 3. Show that $(P \to R) \wedge (Q \to R)$ is equivalent to $(P \vee Q) \to R$.

Exercise 4.

a. State the Principle of Mathematical Induction.

b. The *Well-Ordering Principle* states that *every non-empty subset of* $\mathbb{N}$ *has a least element.* Use the Well-Ordering Principle to prove the Principle of Mathematical Induction. [*Hint:* Argue by contradiction.]

Exercise 5. Determine the validity of the following arguments.

a. Either Jack is busy or he is sick. If he is busy, then he is exhausted. He is not exhausted. Therefore he is sick.

b. If the Dodgers win, then Los Angeles will celebrate, and if the White Sox win, Chicago will celebrate. Either the Dodgers will win or the White Sox will win. However, if the Dodgers win, then Chicago will not celebrate, and if the White Sox win, Los Angeles will not celebrate. So, Chicago will celebrate if and only if Los Angeles does not celebrate.

Exercise 6. Consider the following statement, also known as *Goldbach's conjecture*: *Every even integer* $n \geq 4$ *can be written as the sum of two primes.*

a. Write this statement as an implication.

b. Write the converse and contrapositive of this statement.

c. Negate this statement.

Exercise 7. Prove that there is no pair of positive integers x and y so that $x^2 - y^2 = 10$.

Exercise 8. Let $n \in \mathbb{N}$. Conjecture and prove a closed form expression for

$$\sum_{i=1}^{n} \frac{2}{i(i+2)}.$$

Exercise 9. Let $a, b \in \mathbb{R}$. Prove that for all $n \in \mathbb{N}$, $(ab)^n = a^n b^n$.

Exercise 10. The *symmetric difference* of two sets A and B is defined to be $A \Delta B = (A - B) \cup (B - A)$. Prove that that symmetric difference is associative, that is, show that $A \Delta (B \Delta C) = (A \Delta B) \Delta C$ for all sets A, B and C.

Exercise 11.[Characterization of Union] Given sets A and B, let X be a set with the following properties:

i. $A \subseteq X$ and $B \subseteq X$;

ii. For any set Y, if $A \subset Y$ and $B \subset Y$, then $X \subset Y$.

Show that $X = A \cup B$. Can you formulate a similar characterization for $A \cap B$?

Exercise 12. Let $A = \mathbb{R}^2 - \{(0,0)\}$. Define a relation on A by $(a, b) \sim (c, d)$ if and only if there exists $\lambda \in \mathbb{R} - \{0\}$ so that $(\lambda a, \lambda b) = (c, d)$. Prove that $\sim$ is an equivalence relation. Give a geometric description of $[(a, b)]$.

Exercise 13. Let A and B be sets. Is it true that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$? Provide a proof or a counterexample.