



INTRO TO ABSTRACT MATH  
FALL 2009

HOMEWORK 4  
DUE SEPTEMBER 11

**Exercise 10.** Let  $P$ ,  $Q$  and  $R$  be statements. Verify the following logical equivalences.

a.  $P \wedge (Q \vee R) \cong (P \wedge Q) \vee (P \wedge R)$

b.  $(\neg P) \rightarrow (R \wedge (\neg R)) \cong P$

c.  $\neg(P \rightarrow Q) \cong P \wedge (\neg Q)$

d.  $P \rightarrow (Q \wedge R) \cong (P \rightarrow Q) \wedge (P \rightarrow R)$

e.  $P \leftrightarrow Q \cong (P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$

**Definition:** Let  $m, n \in \mathbb{Z}$ . We say that  $m$  divides  $n$  if there is a  $k \in \mathbb{Z}$  so that  $n = km$ .

**Exercise 11.** Let  $a, b, c \in \mathbb{Z}$ . Prove that if  $a$  divides both  $b$  and  $c$  then  $a$  divides  $b + c$ .

**Exercise 12.** Let  $a, b, c \in \mathbb{Z}$ . Prove that if  $a$  divides both  $b$  and  $c$  then  $a$  divides  $bc$ . Can you modify your proof to yield a slightly stronger statement?