Exercise 16. Negate the following statement in a meaningful way.

For every $x \in \mathbb{Q}$ there exists $y \in \mathbb{N}$ so that $\sin(xy) = 1$ or $\cos(x + y) = 1/2$.

Exercise 17. Consider the following statement: For every real number $x$, $x^2 \geq 0$. Carefully explain why the following proof of this statement is invalid.

Proof. Suppose not. Then for every real number $x$, $x^2 < 0$. In particular, plugging in $x = 3$ we would get $9 < 0$, which is clearly false. This contradiction shows that for every real number $x$, $x^2 \geq 0$. \qed

Exercise 18. What do the following symbolic statements mean? If you assume that the variables involved all belong to $\mathbb{N}$, are they true or false? What if the variables belong to $\mathbb{Z}$ instead?

- a. $\forall x \exists y (x < y)$
- b. $\exists y \forall x (x < y)$
- c. $\exists x \forall y (x < y)$
- d. $\forall y \exists x (x < y)$

Exercise 19. Negate the following statements in a meaningful way.

- a. The product of two rational numbers is rational.
- b. The product of two irrational numbers is irrational.

Exercise 20. Determine if each of the statements in the previous problem is true or false, and justify your answer. (Hint: Recall that a statement is false if its negation is true.)

Exercise 21.

- a. Explain why the statements $\forall x (P(x) \land Q(x))$ and $(\forall x (P(x))) \land (\forall x (Q(x)))$ should be considered logically equivalent.
- b. Find an example that shows $\exists x (P(x) \land Q(x))$ and $(\exists x (P(x))) \land (\exists x (Q(x)))$ are not logically equivalent.