Intro to Abstract Math
Fall 2009

Homework 6
Due September 18

Exercise 16. Negate the following statement in a meaningful way.

For every $x \in \mathbb{Q}$ *there exists* $y \in \mathbb{N}$ *so that* $\sin(xy) = 1$ *or* $\cos(x + y) = 1/2$.

Exercise 17. Consider the following statement: *For every real number* x, $x^2 \geq 0$. Carefully explain why the following proof of this statement is invalid.

Proof. Suppose not. Then for every real number x, $x^2 < 0$. In particular, plugging in $x = 3$ we would get $9 < 0$, which is clearly false. This contradiction shows that for every real number x, $x^2 \geq 0$. □

Exercise 18. What do the following symbolic statements mean? If you assume that the variables involved all belong to $\mathbb{N}$, are they true or false? What if the variables belong to $\mathbb{Z}$ instead?

a. $\forall x \exists y (x < y)$

b. $\exists y \forall x (x < y)$

c. $\exists x \forall y (x < y)$

d. $\forall y \exists x (x < y)$

Exercise 19. Negate the following statements in a meaningful way.

a. The product of two rational numbers is rational.

b. The product of two irrational numbers is irrational.

Exercise 20. Determine if each of the statements in the previous problem is true or false, and justify your answer. (Hint: Recall that a statement is false if its negation is true.)

Exercise 21.

a. Explain why the statements $\forall x(P(x) \wedge Q(x))$ and $(\forall x(P(x))) \wedge (\forall x(Q(x)))$ should be considered logically equivalent.

b. Find an example that shows $\exists x(P(x) \wedge Q(x))$ and $(\exists x(P(x))) \wedge (\exists x(Q(x)))$ are not logically equivalent.