



INTRO TO ABSTRACT MATH
FALL 2009

HOMEWORK 8
DUE SEPTEMBER 23

Exercise 25. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $b|c$ then $a|c$.

Exercise 26. Prove that for all $n \in \mathbb{N}$, $5^n - 1$ is divisible by 4.

Exercise 27. Assume that n is an integer. Prove that for all $n \geq 2$, n can be written as a product of primes (defined below). [*Hint:* Modify the proof on prime numbers given in class.]

Definition: Let $n \in \mathbb{Z}$. We say that n is a *product of primes* if there exist $r \in \mathbb{N}$ and prime numbers p_1, p_2, \dots, p_r (not necessarily distinct) so that $n = p_1 p_2 \cdots p_r$. Notice that, since we can have $r = 1$, every prime number is a product of primes.