Exercise 25. Verify the following claims made in class.

a. For all $n \in \mathbb{N}$ and all $1 \leq k \leq n$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$ 

b. For all $n, l \in \mathbb{N}$

$$l(l+1) \cdots (l+(n-1)) = n! \binom{n+l-1}{l-1}.$$ 

Exercise 26. Prove that for all $n \in \mathbb{N}$

$$2(\sqrt{n+1} - 1) < 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}.$$ 

Exercise 27. Define a sequence of integers recursively by

$$a_1 = a_2 = 1, \quad a_{n+1} = a_n + 2a_{n-1}, \text{ for } n \geq 2.$$ 

a. Write out the first 8 terms of the sequence.

b. Guess an explicit formula for $a_n$. Hint: Multiply the numbers you found in part (a) by 3.

c. Prove that your formula is correct.