Intro to Abstract Math
Fall 2009

Homework 9
Due September 25

Exercise 25. Verify the following claims made in class.

a. For all $n \in \mathbb{N}$ and all $1 \leq k \leq n$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$

b. For all $n, l \in \mathbb{N}$

$$l(l+1)\cdots(l+(n-1)) = n!\binom{n+l-1}{l-1}.$$

Exercise 26. Prove that for all $n \in \mathbb{N}$

$$2(\sqrt{n+1}-1) < 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}.$$

Exercise 27. Define a sequence of integers recursively by

$$\begin{aligned} a_1 &= a_2 = 1 \\ a_{n+1} &= a_n + 2a_{n-1}, \text{ for } n \geq 2. \end{aligned}$$

a. Write out the first 8 terms of the sequence.

b. Guess an explicit formula for a_n. *Hint:* Multiply the numbers you found in part (**a**) by 3.

c. Prove that your formula is correct.