



INTRO TO ABSTRACT MATH  
FALL 2009

HOMework 14  
DUE OCTOBER 9

**Exercise 40.** Recall the equivalence relation

$$Q = \{(a, b), (c, d) \mid ad - bc = 0\}$$

on  $\mathbb{Z} \times \mathbb{N}$ .

- a. Determine the equivalence class of  $(a, b) \in \mathbb{Z} \times \mathbb{N}$ .
- b. Write out several elements (at least 5) of  $[(1, 2)]$ . Do the same for  $[(3, 4)]$ . Do you see a pattern?

**Exercise 41.** Let  $n \in \mathbb{N}$ ,  $n \geq 2$ . Let  $\equiv_n$  denote the relation on  $\mathbb{Z}$  given by

$$\{(a, b) \mid n \text{ divides } a - b\}.$$

- a. Prove that  $\equiv_n$  is an equivalence relation on  $\mathbb{Z}$ .
- b. Determine the equivalence class of  $a \in \mathbb{Z}$ .
- c. By looking at a few cases with small values of  $n$ , make a guess as to how many equivalence classes there are.