Exercise 42. Let $S \subseteq \mathcal{P}(A)$ be a partition of A. Let $\sim = \{(x, y) \mid (\exists X \in S)(x, y \in X)\} \subseteq A^2$. Show that $\sim$ is an equivalence relation on A.

Exercise 43. Recall the equivalence relation $\equiv_2 = \{(a, b) \mid 2 \text{ divides } a - b\}$ on $\mathbb{Z}$ of Exercise 41.

a. Show that $\mathbb{Z}/{\equiv_2} = \{[0], [1]\}$.

b. Let $X_1 = \{n \in \mathbb{Z} \mid n \text{ is odd}\}$ and $X_2 = \{n \in \mathbb{Z} \mid n \text{ is even}\}$. Show that the partition $\{X_1, X_2\}$ of $\mathbb{Z}$ is the same as the set $\mathbb{Z}/{\equiv_2}$.