Exercise 47. Let $A = \{1, 2, 3\}$ and $B = \{\diamondsuit, \heartsuit\}$. Show that there is no injection $f : A \to B$.

Exercise 48. Let $f : X \to Y$ be a function.

  a. If $A_i \subset X$ for $i \in I$, prove that

$$f\left(\bigcap_{i \in I} A_i\right) \subseteq \bigcap_{i \in I} f(A_i).$$

  Find an example to show that these two sets need not be equal.

  b. If $B_i \subset Y$ for $i \in I$, prove that

$$f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i).$$

Exercise 49. Show that $h : [-3, \infty) \to [1, \infty)$ given by $h(x) = 1 + \sqrt{x + 3}$ is a bijection.

Exercise 50. Let A be a set. Recall the function $g : \mathcal{P}(A) \to \mathcal{P}(A)$ given by $g(X) = A - X$. We have already seen that this function is a surjection. Prove that, in fact, it is a bijection.

Exercise 51. Let A be a non-empty set and let $f : A \to \mathcal{P}(A)$ be any function. Show that f is *not* surjective. *Hint:* Consider $S = \{a \in A \mid a \notin f(a)\} \in \mathcal{P}(A)$.