Exercise 52. Let $f$ and $g$ be functions from $X$ to $Y$. Prove that $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$.

Exercise 53. Let $f : \mathbb{R} \to [-4, \infty)$ be given by $f(x) = (x+1)^2 - 4$ and let $g : [-4, \infty) \to \mathbb{R}$ be given by $g(x) = -1 - \sqrt{4 + x}$.

a. Compute $f \circ g$ and $g \circ f$.

b. Are $f$ and $g$ inverses? Be sure to justify your answer.

Exercise 54. Let $f : [0, 1] \to [0, 1]$ be given by $f(x) = \sqrt{1 - x^2}$, let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = x^3 + 1$ and let $h : \mathbb{R} \to \mathbb{R}$ be given by $h(x) = \sqrt[3]{x - 1}$.

a. Show that $f = f^{-1}$.

b. Show that $g = h^{-1}$.

c. Show that $f$ is not a bijection.

Exercise 55. Let $X = \mathbb{R} - \{3\}$. Define $f : X \to \mathbb{R}$ by $f(x) = x/(x - 3)$.

a. Show that $f$ is not a bijection.

b. Find a set $Y \subseteq \mathbb{R}$ so that the function $g : X \to Y$, given by the same formula as $f$, is a bijection.

c. Find $g^{-1}$. 