Intro to Abstract Math | Homework 18
Fall 2009 | Due October 26

Exercise 52. Let f and g be functions from X to Y. Prove that $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$.

Exercise 53. Let $f : \mathbb{R} \to [-4, \infty)$ be given by $f(x) = (x+1)^2 - 4$ and let $g : [-4, \infty) \to \mathbb{R}$ be given by $g(x) = -1 - \sqrt{4+x}$.

- **a.** Compute $f \circ g$ and $g \circ f$.
- **b.** Are f and g inverses? Be sure to justify your answer.

Exercise 54. Let $f : [0,1] \to [0,1]$ be given by $f(x) = \sqrt{1-x^2}$, let $g : \mathbb{R} \to \mathbb{R}$ be given by $g(x) = x^3 + 1$ and let $h : \mathbb{R} \to \mathbb{R}$ be given by $h(x) = \sqrt[3]{x-1}$.

- **a.** Show that $f = f^{-1}$.
- **b.** Show that $g = h^{-1}$.

Exercise 55. Let $X = \mathbb{R} - \{3\}$. Define $f : X \to \mathbb{R}$ by $f(x) = x/(x-3)$.

- **a.** Show that f is not a bijection.
- **b.** Find a set $Y \subseteq \mathbb{R}$ so that the function $g : X \to Y$, given by the same formula as f, is a bijection.
- **c.** Find g^{-1}.