Exercise 56. Let $f : X \to Y$ and $g : Y \to Z$ be functions.

   a. Prove that if f and g are both surjective, then $g \circ f$ is surjective.

   b. Prove that if f and g are both bijective, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. *Hint:* Compose the latter with $g \circ f$ and use the fact that inverses are unique.

Exercise 57.

   a. Show that if $x \geq 0$ then $-4 \leq \dfrac{x^2 - 4}{x^2 + 1} < 1$.

   b. Let $f : [0, \infty) \to [-4, 1)$ be defined by $f(x) = \dfrac{x^2 - 4}{x^2 + 1}$. Show that f is a bijection and find f^{-1}. *Hint:* If you can find the inverse of f, this will prove f is a bijection.

Exercise 58. Let $\mathcal{S}$ denote the collection of all sets. Prove that

$$\approx = \{(X, Y) \in \mathcal{S}^2 \,|\, \text{there is a bijection } f : X \to Y\}$$

is an equivalence relation on $\mathcal{S}$.