Exercise 60. Prove that for all $n \in \mathbb{N}$, $f : I_n \to I_n$ is an injection if and only if it is a surjection.

Exercise 61. Let α and β denote the elements of S_3 with two-row representations

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix},$$

respectively. Prove that every element of S_3 can be obtained as a combination of α and β.

Exercise 62. Let $n \in \mathbb{N}$, $n \geq 2$, and let $i, j \in I_n$. Assume that $i \neq j$. Define $T : I_n \to I_n$ by

$$T(x) = \begin{cases} x & \text{if } x \neq i \text{ and } x \neq j, \\ j & \text{if } x = i, \\ i & \text{if } x = j. \end{cases}$$

a. Prove that $T \in S_n$. The function T is called a *transposition*.

b. What is the two-row description of T?