Intro to Abstract Math — Homework 22

Fall 2009 — Due November 4

Exercise 63. Prove or disprove the following statements.

a. Subtraction is a binary operation on $\mathbb{Z}$.

b. Subtraction is a binary operation on $\mathbb{N}$.

c. Division is a binary operation on $\mathbb{N}$.

d. Division is a binary operation on $\mathbb{Q}$.

Exercise 64. Let

$$\mathrm{Id} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \quad \alpha = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix},$$

$$\gamma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \quad \delta = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \quad \epsilon = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

denote the elements of S_3. We have seen that function composition is a binary operation on S_3. Complete the following "composition table," which gives the results of composing any two elements of S_3. The entry in the x row and y column is xy. The first two rows have been completed for you as an example.

	Id	α	β	γ	δ	ϵ
Id	Id	α	β	γ	δ	ϵ
α	α	Id	δ	ϵ	β	γ
β						
γ						
δ						
ϵ						

Exercise 65. If we let R_θ denote counterclockwise rotation by θ degrees, H denote the flip across the vertical axis, V denote the flip across the horizontal axis, and F_i $(i = 1, 2)$ denote the diagonal flips, then recall that the complete set if symmetries of the square is

$$D_4 = \{R_0, R_{90}, R_{180}, R_{270}, V, H, F_1, F_2\}.$$

Compute the "composition table" for D_4 as you did for S_3 above.