Intro to Abstract Math
Fall 2009

Homework 24
Due November 9

Exercise 69. Write out the Cayley tables for $(\mathbb{Z}_2, +_2)$, $(\mathbb{Z}_3, +_3)$ and $(\mathbb{Z}_4, +_4)$.

Exercise 70. Let $n \in \mathbb{N}$, $n \geq 2$.

a. Let $a \in \mathbb{Z}_n$. Show that if $x \cdot_n a = x$ for all $x \in \mathbb{Z}_n$, then $a = 1$.

b. Show that $(\mathbb{Z}_n, \cdot_n)$ is *never* a group.

Exercise 71. Let $n \in \mathbb{N}$ and $n \geq 2$. Let $\mathbb{N}_n = \mathbb{Z}_n - \{0\}$.

a. For $n = 2, 3, 4, 5, 6$, determine if $\cdot_n$ is a binary operation on $\mathbb{N}_n$.

b. For the n from part **a** for which $\cdot_n$ *is* a binary operation on $\mathbb{N}_n$, write out the Cayley table for $(\mathbb{N}_n, \cdot_n)$.

c. Is $(\mathbb{N}_n, \cdot_n)$ a group for each of the n you used in part **b**?