

Intro to Abstract Math Fall 2009

Homework 25 Due November 13

Exercise 72. Let G be an abelian group and let $a, b \in G$. Prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{N}$. [*Hint:* Use induction.]

Exercise 73. Let $H = \{g : \mathbb{R} \to \mathbb{R} \mid g(x) = ax + b \text{ with } a, b \in \mathbb{R} \text{ and } a \neq 0\}.$

- **a.** Prove that $H \subseteq \operatorname{Aut}(\mathbb{R})$.
- **b.** Prove that H is a subgroup of Aut(\mathbb{R}). [*Note:* Remember that the operation in Aut(\mathbb{R}) is composition.]

Exercise 74. Let *H* and *K* be subgroups of a group *G*. Prove that $H \cap K$ is also a subgroup of *G*.