Exercise 75. Let G be a group with identity element e and let $a \in G$. Suppose that $|a| = n$.

- **a.** Prove that $a^k = e$ if and only if $n|k$. [*Hint:* One implication follows from the laws of exponents. For the other, write $k = qn + r$, where $r \in \mathbb{Z}_n$ is the remainder when k is divided by n. Show that $r \neq 0$ contradicts the fact that $|a| = n$.]
- **b.** Prove that $e, a, a^2, \ldots, a^{n-1}$ are distinct elements of G.
- **c.** Prove that $|\langle a \rangle| = |a|$.
- **d.** Prove that $G = \langle a \rangle$ if and only if $|G| = |a|$.

Exercise 76. Find all the cyclic subgroups of $(\mathbb{Z}_n, +_n)$ for $n = 4, 5, 10, 12$. Identify those a (if there are any) for which $\mathbb{Z}_n = \langle a \rangle$. [*Suggestion:* Just compute $\langle a \rangle$ for every a.]

Exercise 77. Find all the cyclic subgroups of S_3.