Exercise 78. For each pair (a, b), find $\gcd(a, b)$ and express it in the form $ra + sb$ with $r, s \in \mathbb{Z}$.

a. $a = 11$, $b = 3$

b. $a = 42$, $b = 77$

c. $a = 420$, $b = 288$

Exercise 79. Let $n \in \mathbb{N}$, $n \geq 2$ and let $a \in \mathbb{Z}_n$. Prove that if $\gcd(a, n) = 1$ then there is a $b \in \mathbb{Z}_n$ so that $a \cdot_n b = 1$. [*Hint:* If $\gcd(a, n) = 1$ then there are integers r, s so that $ra + sn = 1$.]

Exercise 80. For $n \in \mathbb{N}$, $n \geq 2$, let

$$U(n) = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}.$$

a. Use the result of the previous exercise to show that $(U(n), \cdot_n)$ is a group.

b. Determine whether or not $U(n)$ is cyclic for $n = 8, 9, 10, 11, 12$.