Exercise 81. Let G be a group and $a \in G$. Suppose $|a| = n$. Prove that if $k \in \mathbb{N}$ and $k|n$ then $|a^k| = n/k$.

Exercise 82. Let G be a group, $a \in G$ and $|a| = n$. Recall that for any $k \in \mathbb{Z}$, $\langle a^k \rangle = \langle a^{\gcd(k,n)} \rangle$. Use this to prove that $\langle a^i \rangle = \langle a^j \rangle$ if and only if $\gcd(i, n) = \gcd(j, n)$.

Exercise 83. In the last homework we saw that $U(25)$ is a group under multiplication modulo 25.

  a. List the elements of $U(25)$.

  b. Given that $U(25) = \langle 2 \rangle$, find all the generators of $U(25)$.