INTRO TO ABSTRACT MATH
FALL 2009

HOMEWORK 29
DUE DECEMBER 4

Exercise 84. Show that $\mathbb{Q}$ and $\mathbb{R}$ (under addition) are not cyclic. [*Suggestion:* Argue by contradiction. If a were a generator, what would need to be true about $a/2$?]

Exercise 85. Let G, H and K be groups.

a. Show that if $\varphi : G \to H$ is an isomorphism, then $\varphi^{-1} : H \to G$ is an isomorphism.

b. Show that if $\varphi : G \to H$ and $\psi : H \to K$ are isomorphisms, then $\psi \circ \varphi : G \to K$ is an isomorphism.

c. Show that the notion of isomorphism is an equivalence relation on the collection of all groups.

Exercise 86. Show that no two of $\mathbb{Z}_8$, $U(16)$ and D_4 are isomorphic. Remember, the operation in $\mathbb{Z}_8$ is addition mod 8, and the operation in $U(16)$ is multiplication mod 16.