



INTRO TO ABSTRACT MATH
FALL 2009

HOMEWORK 30
DUE DECEMBER 7

Exercise 87. Let $a \in \mathbb{Q}$, $a \neq 0$. Prove that the function $f : \mathbb{Q} \rightarrow \mathbb{Q}$ given by $f(x) = ax$ is an isomorphism. Conversely, show that if $g : \mathbb{Q} \rightarrow \mathbb{Q}$ is an isomorphism then $g(x) = ax$ for some nonzero $a \in \mathbb{Q}$.

Exercise 88. Let X and Y be (nonempty) sets and let $f : X \rightarrow Y$ be a bijection. Prove that X is countable if and only if Y is countable. [*Suggestion:* Consider the finite and infinite cases separately.]