Exercise 87. Let $a \in \mathbb{Q}, a \neq 0$. Prove that the function $f : \mathbb{Q} \to \mathbb{Q}$ given by $f(x) = ax$ is an isomorphism. Conversely, show that if $g : \mathbb{Q} \to \mathbb{Q}$ is an isomorphism then $g(x) = ax$ for some nonzero $a \in \mathbb{Q}$.

Exercise 88. Let $X$ and $Y$ be (nonempty) sets and let $f : X \to Y$ be a bijection. Prove that $X$ is countable if and only if $Y$ is countable. [Suggestion: Consider the finite and infinite cases separately.]