Isomorphisms

Exercise 1. Let $n \in Z^+$, $n \geq 2$ and

$$G = \{\sigma \in S_n \mid \sigma(n) = n\}.$$

You proved in assignment 5 that $G \leq S_n$. Now show that $G \cong S_{n-1}$.

Exercise 2. Let G be the smallest subgroup of $\mathrm{GL}_2(\mathbb{R})$ containing the matrices

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Show that $G \cong D_4$.

Exercise 3. Let G be the smallest subgroup of $\mathrm{GL}_2(\mathbb{C})$ containing the matrices

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

Show that $|G| = 8$ but $G \ncong \mathbb{Z}_8$ and $G \ncong D_4$. G is called the *quaternion group of order 8*.

Exercise 4. Show that $\mathbb{Z}$ is isomorphic to all of its nontrivial proper subgroups.