Modern Algebra 1 | Homework 1.1
Spring 2010 | Due January 20

Exercise 1. Let G be a nonempty set with a binary operation. We say G is a *monoid* if: (i) $a(bc) = (ab)c$ for all $a, b, c \in G$ and (ii) there is an $e \in G$ (called an *identity*) so that $ae = ea = a$ for all $a \in G$. Note that in particular every group is a monoid.

Which of the following sets with binary operations are monoids? Be sure to justify your answers!

a. $\mathbb{N}$ with multiplication.

b. $\mathbb{N}$ with addition.

c. $\mathbb{Q}^\times$ with division.

d. The set P of 2×2 matrices with positive real entries, and matrix multiplication.

Exercise 2. Prove that the identity element in any monoid (and hence any group) is unique. [*Hint:* If you have two identities, what happens when you multiply them together?]

Exercise 3. Prove that matrix multiplication is an associative operation on $M_2(\mathbb{R})$.