MODERN ALGEBRA 1 | HOMEWORK 1.2
SPRING 2010 | DUE JANUARY 20

Exercise 4. Let G be a group. Use induction to prove that if $a_1, a_2, \ldots, a_n \in G$ then $(a_1 a_2 \cdots a_n)^{-1} = a_n^{-1} \cdots a_2^{-1} a_1^{-1}$.

Exercise 5. Prove that $|S_n| = n!$ and that S_n is non-abelian if $n \geq 3$.

Exercise 6. Find an element $\sigma \in S_5$ so that $\sigma^6 = \epsilon$ (the identity), but $\sigma^i \neq \epsilon$ for $i = 1, 2, 3, 4, 5$.

Exercise 7. Let G be a nonempty set with an associative binary operation (such an object is called a *semigroup*). Prove that G is a group if and only if for every $a, b \in G$ the equations $ax = b$ and $ya = b$ have unique solutions $x, y \in G$. [*Hint:* Use the "one-sided" theorem proven in class.]