Modern Algebra 1 — Homework 2.1
Spring 2010 — Due January 27

Exercise 1. Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 13 & 2 & 15 & 14 & 10 & 6 & 12 & 3 & 4 & 1 & 7 & 9 & 5 & 11 & 8 \end{pmatrix}$$

and let

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ 14 & 9 & 10 & 2 & 12 & 6 & 5 & 11 & 15 & 3 & 8 & 7 & 4 & 1 & 13 \end{pmatrix}.$$

Find the cycle decompositions of each of the following permutations: σ, τ, $\sigma\tau$, $\tau\sigma$, and $\tau^2\sigma$.

Exercise 2. For each of the permutations α whose cycle decompositions you computed in the preceding exercise, determine the least $n \in \mathbb{N}$ so that $\alpha^n = (1)$.

Exercise 3.

a. If $\tau = (1\,2)(3\,4)(5\,6)(7\,8)(9\,10)$ determine whether there is an n-cycle σ ($n \geq 10$) with $\tau = \sigma^k$ for some integer k.

b. If $\tau = (1\,2)(3\,4\,5)$ determine whether there is an n-cycle σ ($n \geq 5$) with $\tau = \sigma^k$ for some integer k.

Exercise 4. Prove that $\mathrm{Sym}(\mathbb{N})$ is infinite (do not say that $\infty! = \infty$).