Modern Algebra 1 — Homework 2.2

Spring 2010 — Due January 27

Exercise 5. Let G be a group and $x, y \in G$. Prove that if $xy = yx^{-1}$ then $x^n y = yx^{-n}$ for all $n \in \mathbb{Z}$. [*Note:* Induction can be used to prove this for all $n \in \mathbb{N}$. Don't forget to deal with the negative integers as well.]

Exercise 6. Write out the Cayley (multiplication) table for D_3, expressing every element in the form $f^i r^j$.

Exercise 7. Show directly (i.e. without arguing geometrically) that every element of D_n of the form fr^j is its own inverse.

Exercise 8. Show that if $x \in D_n$ is not a rotation then $rx = xr^{-1}$.

Exercise 9. If $n = 2k$ and $z = r^k$, show that z is the only nonidentity element of D_n that commutes with every other element.