Modern Algebra 1
Spring 2010

Homework 3.1
Due February 3

Exercise 1. What is the largest possible order of an element of S_7?

Exercise 2. Let $\sigma \in S_n$ and let $i \in \{1, 2, \ldots, n\}$. We say that i is a *fixed point* of σ if $\sigma(i) = i$. Prove that if σ is a cycle containing i and i is a fixed point of σ^k, then the length of σ divides k.

Exercise 3. Let G be a group and let $a, b \in G$ satisfy $ab = ba$. If $|a| = m$ and $|b| = n$ show that $|ab|$ divides the least common multiple of m and n. Give an example of this situation in which $|ab|$ is not equal to the least common multiple.

Exercise 4. Let G be a group in which every element has order less than or equal to 2. Show that G is abelian. [*Hint:* Consider $(ab)^2$ for arbitrary $a, b \in G$.]

Exercise 5. The *exponent* of a group G is the smallest $n \in \mathbb{N}$ so that $a^n = e$ for all $a \in G$.

- **a.** Show that the exponent of G is the least common multiple of the orders of the elements in G.
- **b.** Find the exponents of S_3, S_4 and S_5.