Exercise 1. What is the largest possible order of an element of $S_7$?

Exercise 2. Let $\sigma \in S_n$ and let $i \in \{1, 2, \ldots, n\}$. We say that $i$ is a fixed point of $\sigma$ if $\sigma(i) = i$. Prove that if $\sigma$ is a cycle containing $i$ and $i$ is a fixed point of $\sigma^k$, then the length of $\sigma$ divides $k$.

Exercise 3. Let $G$ be a group and let $a, b \in G$ satisfy $ab = ba$. If $|a| = m$ and $|b| = n$ show that $|ab|$ divides the least common multiple of $m$ and $n$. Give an example of this situation in which $|ab|$ is not equal to the least common multiple.

Exercise 4. Let $G$ be a group in which every element has order less than or equal to 2. Show that $G$ is abelian. [Hint: Consider $(ab)^2$ for arbitrary $a, b \in G$.]

Exercise 5. The exponent of a group $G$ is the smallest $n \in \mathbb{N}$ so that $a^n = e$ for all $a \in G$.

  a. Show that the exponent of $G$ is the least common multiple of the orders of the elements in $G$.

  b. Find the exponents of $S_3, S_4$ and $S_5$. 