Modern Algebra 1
Spring 2010

Homework 4.1
Due February 10

Exercise 1. Recall the group $Q = \langle A, B \rangle$ where $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ (this group is known as the *quaternion group*, by the way). Find all the subgroups of Q and draw the subgroup lattice for Q. [*Note:* It may be useful to recall that $Q = \{\pm I, \pm A, \pm B, \pm AB\}$, where I is the 2×2 identity matrix.]

Exercise 2. Determine all of the subgroups of $\mathbb{Z}$ that contain $\langle 90 \rangle$ and draw a lattice of these subgroups. This lattice is actually "isomorphic" to the subgroup lattice of $\mathbb{Z}_{90}$, as we'll see later.

Exercise 3. How does Lang's definition of an *ideal* in $\mathbb{Z}$ (c.f. section I.3) compare with the notion of a subgroup of $\mathbb{Z}$? How does his Theorem 3.1 compare to the theorem on the subgroups of $\mathbb{Z}$ that we proved in class?

Exercise 4. Show that $\mathbb{Q}$ (with addition) is not *finitely generated*, i.e. given any finite set $r_1, r_2, \ldots, r_n \in \mathbb{Q}$ then $\langle r_1, r_2, \ldots, r_n \rangle \neq \mathbb{Q}$. [*Suggestion:* Show that there are only a limited number of denominators that can be obtained from integral linear combinations of $r_1, r_2, \ldots, r_n$.]