Modern Algebra 1 — Homework 4.2
Spring 2010 — Due February 10

Exercise 5. Let $f : G \to H$ be a group homomorphism.

a. Let $a \in G$. Prove that $f(a^n) = f(a)^n$ for all $n \in \mathbb{Z}$.

b. Let $a \in G$. Prove that $f(\langle a \rangle) = \langle f(a) \rangle$.

c. Let $a \in G$. Prove that if a has finite order then so does $f(a)$. How are their orders related? Justify your answer.

d. Let $K \leq H$. Prove that $f^{-1}(K) \leq G$.

Exercise 6. Let G be a group, $a \in G$ and $f : \mathbb{Z} \to \langle a \rangle$ be the homomorphism given by $f(n) = a^n$. If a has finite order prove that $\ker f = \langle |a| \rangle$.

Exercise 7. Let G be an abelian group and let $n \in \mathbb{Z}$. Define $f : G \to G$ by $f(x) = x^n$.

a. Prove that f is a homomorphism.

b. Compute $\ker f$.

c. Use f to show that $\{y \in G \mid y = x^n \text{ for some } x \in G\}$ is a subgroup of G