Modern Algebra 1 — Homework 5.2

Spring 2010 — Due February 17

Exercise 5. Let G be a group and for $a, b \in G$ define $a \sim b$ if and only if there is an $x \in G$ so that $xax^{-1} = b$. Prove that $\sim$ is an equivalence relation on G.

Exercise 6. Let $n \geq 2$ and $\sigma \in S_n$. Prove that $\sigma(a_1\, a_2\, \cdots\, a_k)\sigma^{-1} = (\sigma(a_1)\, \sigma(a_2)\, \cdots\, \sigma(a_k))$. Use this to prove that two elements of S_n are conjugate if and only if their cycle decompositions have the same "cycle structure."

Exercise 7. For each of the following pairs σ, τ of permutations, find a permutation γ so that $\gamma\sigma\gamma^{-1} = \tau$.

a. $\sigma = (15)(23)$, $\tau = (13)(45)$

b. $\sigma = (123)(46)(78)$, $\tau = (135)(27)(48)$

c. $\sigma = (123)(456)$, $\tau = (135)(246)$

Exercise 8. Compute $Z(Q)$. [See homework 4.1 for the definition of Q.]