Modern Algebra 1 | Homework 6.1
Spring 2010 | Due February 24

Exercise 1. Let G be a group, let $a \in G$ and let $T_a : G \to G$ be defined by $T_a(x) = ax$. Prove that T_a is a bijection. [*Suggestion:* Find T_a^{-1}.]

Exercise 2. Let $Q = \langle A, B \rangle = \{\pm I, \pm A, \pm B, \pm AB\}$ (c.f. Homework 4.1, Exercise 1). Identify Q with $\{1, 2, \ldots, 8\}$ as follows:

$$\begin{array}{cccccccc} I & A & -I & -A & B & AB & -B & -AB \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$

Under this identification, left-multiplication in Q by an element of Q yields a permutation in S_8, and according to (the proof of) Cayley's theorem the resulting collection of permutations is a subgroup of S_8 isomorphic to Q. Find all of the elements of this subgroup.