Modern Algebra 1 — Homework 6.2
Spring 2010 — Due February 24

Exercise 3. Let $f : A \to B$ be a bijection of sets. Prove that the function $F : \mathrm{Sym}(A) \to \mathrm{Sym}(B)$ given by $F(g) = f \circ g \circ f^{-1}$ is a group isomorphism. Conclude that if A has size n then $\mathrm{Sym}(A) \cong S_n$.

Exercise 4. Let G be a group and let H be a subgroup. Given $x, y \in G$ we say that x is *congruent to y modulo H* provided $x^{-1}y \in H$. If x and y are congruent modulo H then we write $x \equiv y \pmod{H}$.

a. Prove that congruence modulo H is an equivalence relation on G.

b. Determine the equivalence class of $x \in G$.

c. If H is the trivial subgroup, when are two elements of G congruent modulo H?

Exercise 5. Show that the distinct cosets of $H = \mathrm{SL}_2(\mathbb{R})$ in $G = \mathrm{GL}_2(\mathbb{R})$ are given by

$$\begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix} H$$

with $x \in \mathbb{R}$.