MODERN ALGEBRA 1
SPRING 2010

HOMEWORK 6.3
DUE FEBRUARY 24

Exercise 6. Let G be a group and $H \leq G$. What can you say about H if $[G : H] = 1$? What about if G is finite and $[G : H] = |G|$?

Exercise 7. Use Lagrange's Theorem to prove that the index is *multiplicative in towers*. That is, if G is a finite group and $K \leq H \leq G$ then $[G : K] = [G : H][H : K]$. [*Note:* This equality is also true if G is infinite and the subgroups are of finite index, but you don't have to prove that.]

Exercise 8. Let G be a finite group and $H \leq G$. Prove that if $[G : H]$ is prime then H is maximal (c.f. Homework 3.3).