Modern Algebra 1 — Homework 7.2
Spring 2010 — Due March 10

Exercise 4. Let $f : G \to H$ be a homomorphism of groups.

a. If $K \lhd H$ prove that $f^{-1}(K) \lhd G$.

b. If $K \lhd G$ and f is surjective prove that $f(K) \lhd H$.

Exercise 5. Let G be a group and $H, K \leq G$.

a. Prove that if $H \lhd G$ then $HK = KH$ and $HK \leq G$.

b. Prove that if G is abelian then $HK \leq G$. [*Suggestion:* Use part **a**.]

Exercise 6. To what familiar group is $D_n/\langle r \rangle$ isomorphic? Be sure to justify your answer.

Exercise 7. Let G be an abelian group and $H \leq G$. Prove that G/H is abelian.

Exercise 8. Show that $\mathbb{Q}/\mathbb{Z}$ is infinite, but that every element in it has finite order.

Exercise 9. Suppose that $G = \langle a \rangle$ is a cyclic group and $H \leq G$. Prove that G/H is cyclic. What is it generated by? [*Suggestion:* Use the canonical surjection $f : G \to G/H$ and exercise 5b from homework 4.]