MODERN ALGEBRA 1 — HOMEWORK 8.1
SPRING 2010 — DUE MARCH 24

Exercise 1. Let G be a group and suppose that $|G| = p$, a prime number. Prove that $G \cong \mathbb{Z}/\langle p \rangle$. Find, with proof, every subgroup of G.

Exercise 2. Suppose that G is a finite group and that $f : G \to H$ is a group homomorphism. Use Lagrange's Theorem and the First Isomorphism Theorem to prove that $|G| = |\ker f||\operatorname{Im} f|$.

Exercise 3. Let G be a group and for $a \in G$ let $c_a : G \to G$ denote the automorphism of G given by $c_a(x) = axa^{-1}$. Recall that $\operatorname{Inn}(G) = \{c_a \,|\, a \in G\}$ and $Z(G) = \{x \in G \,|\, xy = yx \text{ for all } y \in G\}$. Use the first isomorphism theorem to prove that $G/Z(G) \cong \operatorname{Inn}(G)$.

Exercise 4. Lang, II.4.28.