MODERN ALGEBRA 1 | HOMEWORK 8.2
SPRING 2010 | DUE MARCH 24

Exercise 5. Let G be a group. Show that if H and K are both normal subgroups of G and $H \cap K = \{e\}$ then $xy = yx$ for all $x \in H$ and $y \in K$. [*Hint:* Consider the element $xyx^{-1}y^{-1}$.]

Exercise 6. Let G and H be groups and let $J \leq G$ and $K \leq H$.

- **a.** Prove that $J \times K \leq G \times H$.
- **b.** If $J \lhd G$ and $K \lhd H$ then $J \times K \lhd G \times H$ and $(G \times H)/(J \times K) \cong G/J \times H/K$. [*Suggestion:* Use the first isomorphism theorem.]
- **c.** Is every subgroup of $G \times H$ of the form $J \times K$?

Exercise 7. If m and n are *not* relatively prime show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is *not* cyclic.

Exercise 8. Lang, II.4.29.