Exercise 5. Let $G$ be a group. Show that if $H$ and $K$ are both normal subgroups of $G$ and $H \cap K = \{e\}$ then $xy = yx$ for all $x \in H$ and $y \in K$. [Hint: Consider the element $xyx^{-1}y^{-1}$.]

Exercise 6. Let $G$ and $H$ be groups and let $J \leq G$ and $K \leq H$.

a. Prove that $J \times K \leq G \times H$.

b. If $J \triangleleft G$ and $K \triangleleft H$ then $J \times K \triangleleft G \times H$ and $(G \times H)/(J \times K) \cong G/J \times H/K$.

[Suggestion: Use the first isomorphism theorem.]

c. Is every subgroup of $G \times H$ of the form $J \times K$?

Exercise 7. If $m$ and $n$ are not relatively prime show that $\mathbb{Z}_m \times \mathbb{Z}_n$ is not cyclic.

Exercise 8. Lang, II.4.29.