Exercise 10. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be given by $f(x, y) = 2x - 3y$.

a. Prove that f is a surjective homomorphism.

b. Find $\ker f$ and describe it and its cosets geometrically.

c. The First Isomorphism Theorem implies that f induces an isomorphism $\overline{f} : (\mathbb{R} \times \mathbb{R})/\ker f \to \mathbb{R}$. Describe this isomorphism geometrically.

Exercise 11. Let G be a finite group, let $H \leq G$ and let $K \lhd G$. Prove that if $|H|$ is relatively prime to $[G : K]$ then $H \leq K$. [*Hint:* Given $a \in H$, by considering the orders of a in H and aK in G/K, show that $\langle a \rangle \leq K$.]

Exercise 12. Lang, II.4.30.