



MODERN ALGEBRA 1
SPRING 2010

HOMEWORK 11.1
DUE APRIL 14

Exercise 1. Express the following permutations as products of transpositions.

a. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 5 & 7 & 1 & 4 & 6 & 10 & 9 & 2 & 8 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 1 & 5 & 6 & 7 & 2 & 8 & 9 & 10 & 3 & 11 & 12 & 4 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 7 & 6 & 5 & 1 & 2 & 3 & 4 & 9 \end{pmatrix}$

Exercise 2. Let $A = \begin{pmatrix} 4 & 1 \\ 3 & 3 \end{pmatrix}$. Determine if A is a unit in $M_2(\mathbb{Z}_n)$ for $n = 1, 2, 3, \dots, 10$ and find its inverse.

Exercise 3. Use the correspondence principle to draw the subgroup lattice of \mathbb{Z}_{84} . [*Hint:* Start with the lattice of subgroups of \mathbb{Z} that contain $\langle 84 \rangle$.]