Modern Algebra 1 | Homework 12.2
Spring 2010 | Due April 21

**Exercise 3.** Let $f : G \to H$ be a homomorphism of abelian groups. We say that $f$ *splits* if there is a homomorphism $g : H \to G$ so that $f \circ g = \mathrm{Id}_H$. Show that if $f$ splits then $G \cong \ker f \times \operatorname{Im} f$. [*Hint:* Define $F : \ker f \times \operatorname{Im} f \to G$ by $F(x, y) = x + g(y)$. Prove that $F$ is an isomorphism.]

**Exercise 4.** Let $G$ be a finite abelian group that is not cyclic. Show that there is a prime $p$ that divides $|G|$ so that $G$ contains a subgroup isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$.