MODERN ALGEBRA 1
SPRING 2010

HOMEWORK 12.3
DUE APRIL 21

Exercise 5. Let G be an (additive) abelian group and let $m \in \mathbb{Z}$.

a. Prove that the function $f_m : G \to G$ given by $f_m(x) = mx$ is a homomorphism.

b. Use part **a** to show that G_m and $mG = \{mx \,|\, x \in G\}$ are both subgroups of G.

c. If G is finite, under what conditions on m and $|G|$ is f_m injective?

Exercise 6. If p is a prime and G is an abelian p-group, show that the homomorphism f_p of the preceding exercise is *not* onto. Conclude that $|pG| < |G|$.

Exercise 7. If p is a prime and $n \in \mathbb{N}$ prove that $p\mathbb{Z}_{p^n} \cong \mathbb{Z}_{p^{n-1}}$. [*Suggestion:* Use the homomorphism f_p of exercise 5 and the fact that subgroups of cyclic groups are cyclic.]