Differential Equations
Spring 2011

Assignment 1.1
Due January 21

Exercise 1. Determine the order of each of the following differential equations, and state whether they are linear or nonlinear.

a. $t^2 \dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin t$

b. $(1+y)\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + y = e^t$

c. $\dfrac{d^4y}{dt^4} + \dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + y = 1$

d. $\dfrac{dy}{dt} + ty^2 = 0$

e. $\dfrac{d^2y}{dt^2} + \sin(t+y) = \sin t$

f. $\dfrac{d^3y}{dt^3} + t\dfrac{dy}{dt} + (\cos^2 t)y = t^3$

Exercise 2. Use a computer to draw a direction field for each of the following differential equations. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency.

a. $y' = y(4-y)$

b. $y' = y(y-2)^2$

c. $y' = te^{-2t} - 2y$

d. $y' = 3\sin t + 1 + y$

e. $(1+t^2)y' + 4ty = (1+t^2)^{-2}$

f. $2y' + y = 3t^2$

Exercise 3. A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time.

Exercise 4. Determine all values of r for which the differential equation $t^2y'' - 4ty' + 4y = 0$ has solutions of the form $y = t^r$ for $t > 0$.

Exercise 5. Use the chain rule to show that the change of variable $x = \ln t$ transforms the differential equation $t^2\dfrac{d^2y}{dt^2} + at\dfrac{dy}{dt} + by = 0$, $t > 0$, into $\dfrac{d^2y}{dx^2} + (a-1)\dfrac{dy}{dx} + by = 0$.