Exercise 1. Consider the initial value problem

$$y' + \frac{2}{3}y = 1 - \frac{1}{2}t, \qquad y(0) = y_0.$$

Find the value of y_0 for which the solution touches, but does not cross, the t-axis.

Exercise 2. Consider the initial value problem

$$y' - \frac{2}{3}y = 3t + 2e^t, \qquad y(0) = y_0.$$

Find the value of y_0 that separates solutions that grow positively as $t \to \infty$ from those that grow negatively. How does the solution that corresponds to this critical value of y_0 behave as $t \to \infty$?

Exercise 3. Construct a first order linear differential equation all of whose solutions approach the curve $y = 4 - t^2$ as $t \to \infty$. Solve your differential equation and confirm that the solutions do indeed have this property.

Exercise 4. Solve the initial value problem

$$y' = \frac{1 + 3x^2}{3y^2 - 6y}, \qquad y(0) = 1$$

and determine the interval in which the solution is valid. [*Hint:* To find the interval of definition, look for points where the solution has vertical tangents.]

Exercise 5. Consider the initial value problem

$$y' = \frac{ty(4 - y)}{1 + t}, \qquad y(0) = y_0 > 0.$$

a. Determine how the solution behaves as $t \to \infty$.

b. If $y_0 = 2$, find the time T at which the solution first reaches the value 3.99.

c. Find the range of initial values for which the solution lies in the interval $3.99 < y < 4.01$ by the time $t = 2$.