Differential Equations
Spring 2011

Assignment 5.1
Due February 16

We have seen that if $p(t)$ and $q(t)$ are continuous on an open interval I on which $q(t)$ is positive, in order for a change of variable of the form $x = x(t)$ to transform the differential equation $\dfrac{d^2y}{dt^2} + p(t)\dfrac{dy}{dt} + q(t)y = 0$ into one with constant coefficients, it is necessary and sufficient that

$$\frac{q'(t) + 2p(t)q(t)}{q(t)^{3/2}} \text{ is constant.} \tag{1}$$

Moreover, in this case the required change of variables is

$$x = \int q(t)^{1/2}\,dt.$$

Exercise 1. Show that every Euler equation $t^2y'' + aty' + by = 0$ $(t > 0)$ satisfies (1). You may assume that $b > 0$.

Exercise 2. 3.3.36

Exercise 3. 3.3.40

Exercise 4. 3.4.41

Exercise 5. 3.3.45

Exercise 6. Determine every possible function $p(t)$ for which $y'' + p(t)y' + ty = 0$ $(t > 0)$ can be transformed into an equation with constant coefficients, and determine the required change of variables.