Differential Equations
Spring 2011

Assignment 9.1
Due March 25

Exercise 1. Consider the equation

$$(1 + x^2)y'' + Axy' + By = 0 \tag{1}$$

where A and B are arbitrary (real) constants.

a. Show that (1) has analytic solutions centered at $x_0 = 0$ with radii of convergence at least 1.

b. Show that if

$$y = \sum_{n=0}^{\infty} a_n x^n$$

is a solution to (1) then its coefficients satisfy

$$a_{n+2} = \frac{-(n^2 + (A-1)n + B)}{(n+2)(n+1)} a_n \tag{2}$$

for $n \geq 0$.

c. Let y_1 denote the solution with $a_0 = 1$ and $a_1 = 0$. If $A = 1$ and $B = -1$, find a closed form expression for a_n.

d. Repeat part (**c**) for the solution y_2 that satisfies $a_0 = 0$, $a_1 = 1$.

e. Find values of A and B so that (1) has only rational solutions. State these solutions explicitly. [*Suggestion:* Adjust A and B so that the rational function in n appearing in (2) "goes away."]