Number Theory II
Fall 2010

Assignment 12.1
Due November 17

Exercise 1. Let $k \in \mathbb{N}$ with $k \geq 2$ and suppose that a is an arithmetic function with period k. If $\sum_{n=1}^{k} a(n) = 0$, prove that the function $A(x) = \sum_{n \leq x} a(n)$ has period k. Deduce that $A(x)$ is bounded.

Exercise 2. Recall that if $k \in \mathbb{N}$, $k \geq 2$, and χ_0 is the principal character mod k then for $s > 1$ we defined

$$L(s, \chi_0) = \sum_{\substack{n=1 \\ (n,k)=1}}^{\infty} \frac{1}{n^s}.$$

a. Use Theorems 2.18 and 3.2 to prove that for $s > 0$, $s \neq 1$, we have

$$\sum_{\substack{n \leq x \\ (n,k)=1}} \frac{1}{n^s} = \frac{\varphi(k)}{k} \frac{x^{1-s}}{1-s} + \zeta(s) \prod_{p|k} \left(1 - p^{-s}\right) + O(x^{-s}).$$

b. Use part (**a**) to deduce that

$$L(s, \chi_0) = \zeta(s) \prod_{p|k} \left(1 - p^{-s}\right)$$

for $s > 1$.