Exercise 1. Let $k \in \mathbb{N}$ with $k \geq 2$ and let χ be a Dirichlet character mod k.

a. Prove that the series

$$\sum_{n=2}^{\infty} \frac{1}{n} \sum_{p} \frac{\chi(p^n)}{p^{ns}}$$

converges absolutely for all $s > 1/2$. [*Suggestion:* Use geometric series.]

b. Show that if $s \geq 1$ then the (absolute value of the) series in part **a** is bounded by

$$\sum_{m=2}^{\infty} \frac{1}{m(m-1)}.$$

Exercise 2. Find a modulus k and a Dirichlet character χ mod k so that

$$\frac{1}{2} L(1,\chi) = \frac{1}{1 \cdot 3} + \frac{1}{5 \cdot 7} + \frac{1}{9 \cdot 11} + \cdots .$$