Number Theory II
Fall 2010

Assignment 13.1
Due December 1

Exercise 1. Let f and g be continuously differentiable real-valued functions on the interval $[a, b]$. Define the complex-valued function F on $[a, b]$ by $F(x) = f(x) + ig(x)$. Show that $F(x) = F(a) + O(x - a)$ for all $x \in [a, b]$. Conclude that if $F(a) = 0$ then there is a constant C so that $|F(x)| \leq C(x - a)$ for all $x \in [a, b]$. [*Suggestion:* Apply the mean value theorem to f and g on the interval $[a, x]$.]

Exercise 2. Let χ be a real-valued Dirichlet character. Prove that $\zeta(s)L(s, \chi) \geq \zeta(2s)$ for $s > 1$. [*Suggestion:* The series defining $\zeta(s)$ and $L(s, \chi)$ are absolutely convergent for $s > 1$ and can be multiplied term by term. Do this, and then use Theorem 6.19.]