



NUMBER THEORY II
FALL 2010

ASSIGNMENT 8.1
DUE OCTOBER 20

Exercise 1. Let $d_n = \text{lcm}(1, 2, 3, \dots, n)$, let p be a prime, and let $a \in \mathbb{N}$. Prove that p^a divides d_n if and only if $p^a \leq n$.

Exercise 2. Use Abel summation to prove that for $x \geq 2$

$$\sum_{p \leq x} \frac{1}{p} = \frac{\pi(x)}{x} + \int_2^x \frac{\pi(t)}{t^2} dt.$$

Use this and the bounds for $\pi(x)$ that we have proven in class (or the prime number theorem, if you want to go that far) to determine whether $\sum_p \frac{1}{p}$ converges or diverges.