Number Theory II
Fall 2010

Assignment 9.2
Due October 27

Exercise 1. Let f be a real-valued function that is Riemann integrable on every finite subinterval of $[0, \infty)$. Assume further that $\lim_{x\to\infty} f(x) = 0$.

a. Show that for any $\epsilon > 0$ there is an $M > 0$ so that for all $x \geq M$

$$\left|\frac{1}{x}\int_M^x f(t)\,dt\right| < \frac{\epsilon}{2}.$$

b. For M and ϵ as in part **a**, show that there is an $N \geq M$ so that if $x \geq N$ then

$$\left|\frac{1}{x}\int_0^M f(t)\,dt\right| < \frac{\epsilon}{2}.$$

c. Use parts **a** and **b** to conclude that

$$\lim_{x\to\infty}\frac{1}{x}\int_0^x f(t)\,dt = 0.$$

Exercise 2. Chapter 4, 27(a) [*Note:* You may assume that $A(x) = O(x)$, which is actually a consequence of the stated hypotheses: see the first part of the proof of Theorem 4.8.]