



NUMBER THEORY II
FALL 2010

ASSIGNMENT 10.1
DUE NOVEMBER 3

Exercise 1. Let $\mathbb{C}^\times = \mathbb{C} - \{0\}$ and $U = \{z \in \mathbb{C} : |z| = 1\}$.

- a. Show that \mathbb{C}^\times is a group under multiplication.
- b. Show that U is a subgroup of \mathbb{C}^\times .
- c. Show that if G is a subgroup of \mathbb{C}^\times and $G \not\subseteq U$, then $\{|z| : z \in G\}$ is unbounded.
- d. Show that if G is a finite subgroup of \mathbb{C}^\times then $G \subseteq U$.

Exercise 2. Show that every finite subgroup of \mathbb{C}^\times is cyclic. [*Suggestion:* If the subgroup is nontrivial, show the element with the smallest (nonzero) argument is a generator.]